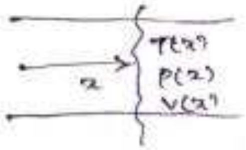
Air breathing propulsion: consumes its oxygen for combustion from atmosphere

Stagnation properties of gas:

For an isentropic flow field with local gas temperature $T(x)$, pressure $p(x)$ and velocity $V(x)$,



$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/\gamma-1}$$

$$= \left\{1 + \frac{\gamma-1}{2} M^2\right\}^{\gamma/\gamma-1}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/\gamma-1} = \left\{1 + \frac{\gamma-1}{2} M^2\right\}^{1/\gamma-1}$$

For an isentropic nozzle, P_0 and T_0 are constant

$$T(x) = \frac{T_0}{1 + \frac{\gamma-1}{2} M^2(x)}$$

$$P(x) = \frac{P_0}{\left\{1 + \frac{\gamma-1}{2} M^2(x)\right\}^{\gamma/\gamma-1}}$$

For adiabatic flow, $\quad h(x) + \frac{V^2(x)}{2} = C_p T(x) + \frac{V^2(x)}{2} = \text{constant}$

$$C_p T(x) + \frac{V^2(x)}{2} = C_p T_0$$

$$T_0 = T(x) + \frac{V^2(x)}{2C_p}$$

$$\frac{V^2/2}{C_v T} = \frac{\gamma(\gamma-1)}{2} M^2$$

$$\frac{V^2}{2\gamma C_v} = \frac{V^2}{2 \frac{C_p}{C_v} C_v} = \frac{V^2}{2C_p} = T \frac{(\gamma-1)}{2} M^2$$

$$T_0 = T(x) + T(x) \frac{\gamma-1}{2} M^2(x)$$

$$\frac{T_0}{T(x)} = 1 + \frac{\gamma-1}{2} M^2(x)$$

Assumptions for Ideal Rocket:

SPATER

(i) The working substance is homogeneous.

(ii) All the species of working fluid are gaseous

(iii) The working substance obeys perfect gas law.

(iv) No heat transfer across the rocket and therefore the flow is adiabat

(v) No appreciable friction and all boundary layer effects are neglect

(vi) No shock waves or discontinuities in the nozzle flow.

(vii) Propellent flow is steady & adiabatic.

(viii) All exhaust gases leaving the rocket have axially directed velocity.

(ix) The gas velocity, pressure, density are all uniform across any section normal to nozzle axis.

(x) chemical equilibrium is established within the rocket chamb and gas composition does not change in the nozzle.

(xi) stored propellents are at their room temp. Cryogenic propellents are at their boiling point.

Real nozzle:

For single, simple nozzle shapes, the temperature and velocities are not uniform over one section and usually hi in central large region and lower in peripheral region.

The Mach number is 1 is a plane at the throat for ideal nozzle. For 2-D flow, it is typically a slightly curved surfac somewhat downstream of the throat. If the velocity distribut for axisymmetric n is known, the average velocity v_2 can be determined, as a function of radius.

$$(v_2)_{average} = \frac{2\pi}{A_2}\int_0^{r_2} v_2 r\,dr$$

Real nozzle has energy losses and energy that is unavailable for conversion into kinetic energy of exhaust gases.

SPATER

1. The divergence of flow in the nozzle exit planes causes a loss, which varies as a function of cosine of the divergence angle.

2. Small chamber or port area cross-sections relative to the throat area or low nozzle contraction ratios A_1/A_t causes pressure losses in the chamber and reduce the thrust and exhaust velocity.

3. Lower flow velocity in the boundary layer or wall friction can reduce effective exhaust gas velocity. 0.5 ~ 1.5%

4. Solid particles or liquid droplets in the gas can cause loss upto 5%.

5. Unsteady combustion and oscillatory flows can cause small loss.

6. chemical reactions in the nozzle flow change gas properties, temperature upto ~~5%~~ 0.5%.

7. There is lower performance during transient pressure operation - start, stop, pulsing.

8. For uncooled nozzle mat., FRP, carbon, the gradual erosion of the throat region increases throat diameter by perhaps 1 to 6% during operation resulting in 0.6% reduction in I_{sp}.

9. Non-uniform gas composition can reduce performance.

10. Using real gas properties can at times changes the gas temp, K and M → loss in performance 0.2 ~ 0.7%

11. Operation at non-optimum nozzle expansion area ratio can reduce thrust and specific impulse. There is no loss if the vehicle flies always at optimum nozzle expansion. If it flies

Mass flow rate per unit area

$$\frac{\dot{m}}{A_e} = \rho V = \frac{P}{R_g T}\cdot V$$

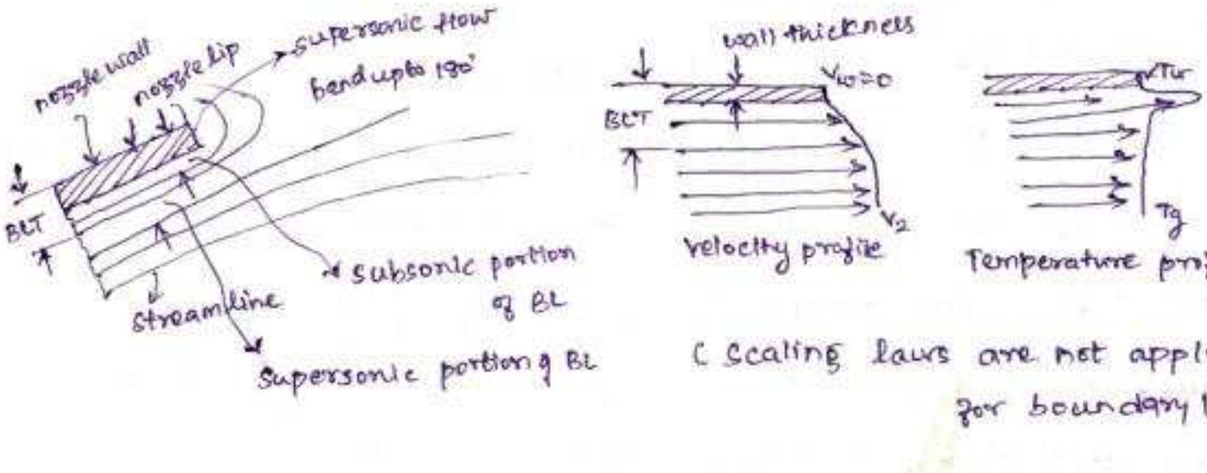
$$= \frac{P}{\gamma R_g T}\cdot \gamma V = \frac{P}{\sqrt{\gamma R_g T}}\cdot \gamma \cdot \frac{V}{\sqrt{\gamma R_g T}}$$

$$= \frac{\gamma P M}{\sqrt{\gamma R_g T}} = \sqrt{\frac{\gamma}{R_g}}\cdot \frac{P}{\sqrt{T}} M$$

$$= \sqrt{\frac{\gamma}{R_g}}\cdot \sqrt{\frac{T_0}{T}}\cdot \frac{P}{P_0}\cdot \frac{P_0}{\sqrt{T_0}} M$$

Boundary layer:

Real nozzles have viscous boundary layers next to the nozzle walls whereas the ~~flow~~ gas velocities are much lower than the free stream velocities in the inviscid flow region.



(Scaling laws are not applic for boundary la

Low velocity flow close to the wall is laminar and subsonic. But in the high velocity regions of boundary lay the flow is supersonic and ~~turbed~~ becomes turbulent.

The local temperature in the part of boundary lay is substantially higher than the free stream temperatur because of the conversion of kinetic energy into thermal energy as the local velocity is slowed down and as heat is created by viscous friction. The layer right next to boundary layer has a profound effect on the overall heat transfer to the nozzle and chamber walls. The high gradients in pressure, temperature or density and changes in local velocity increases the boundary layer.

Performance correction factors:

Energy conversion efficiency: $e = \frac{(v_2)_a^2}{(v_2)^2} = \frac{(v_2)_a^2}{(v_1)_a^2 + c_p(T_1 - T_2)}$

The ratio of K.E. per unit of flow of the actual jet leaving the nozzle to the K.E. per unit of flow of a hypothetical ideal exhaust jet that is supplied with the same working substance at the same initial velocity and state and expands to same exit pressure as the real nozzle.

Velocity correction factor is defined as the square root of energy conversion efficiency.

Discharge correction factor is the ratio of mass flow rate of real rocket to that of ideal rocket that expands to an identical working fluid from the same initial condition to same exit pressure.

$$\zeta_d = \frac{\dot{m}_a \sqrt{KRT_1}}{A_t P_1 K \sqrt{\{2/K+1\}^{K+1/K-1}}}$$

Qn: Design a rocket nozzle to conform follow conditions.

Chamber pressure = 20.4 times atmosphere = 2.068 MPa

Chamber temperature = 2681 K

Mean molecular mass of gas = 21.87

Ideal specific impulse = 230

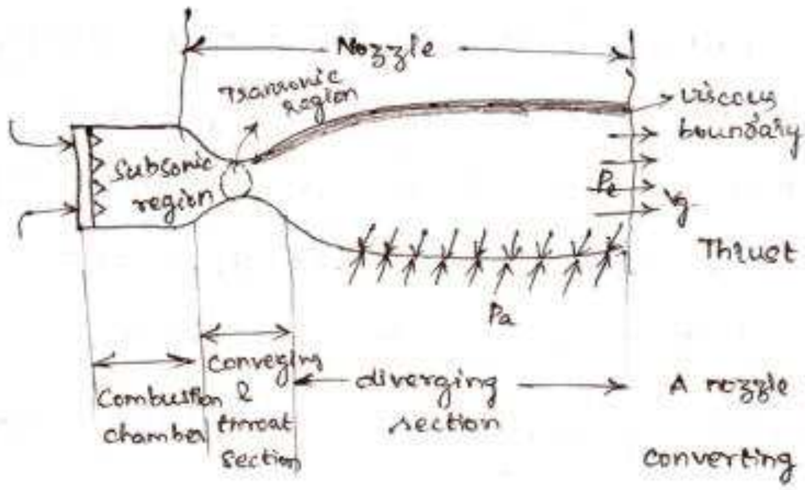
$\gamma = 1.229$; Desired thrust = 1300N

Determine nozzle throat, and exit area, diameters, actual ve
actual specific impulse.

SPATER

Performance parameters of nozzle:

1. chamber pressure
2. Ambient pressure
3. Nozzle expansion area ratio
4. Nozzle shape exit angle
5. Prop. & their comp. or mixture ratios
6. Assumptions



Thrust $= (\dot{M}V_e + P_eA_e) - P_aA_e$

A nozzle generates thrust by converting the thermal energy of hot combustion gases (temp) to kinetic energy (velocity). Max. theorit thrust is achieved in vacuum conditions when the nozzle area is ∞

(Aerospike nozzle, Bell nozzles)

Supersonic nozzle design is the culmination of carefully determined mission needs, established physics and rocket d (rocket dyne), contributions in propulsion are now being applied to the state of art hypersonic flight.

(http://www.k-makris.gr/Rocket Technology/Nozzle Dimensions/Nozzle_De

viscous drag efficiency:

$$\eta_{drag} = 1 - \frac{\Delta C_{f(drag)}}{C_{f(ideal)}}$$

C_f - skin friction co-efficient

chemical K.E. efficiency:

$$\eta_{KiE} = 1 - \frac{\Delta G_{f(OD,R)}}{\Delta G_{f(OD,E)}}$$

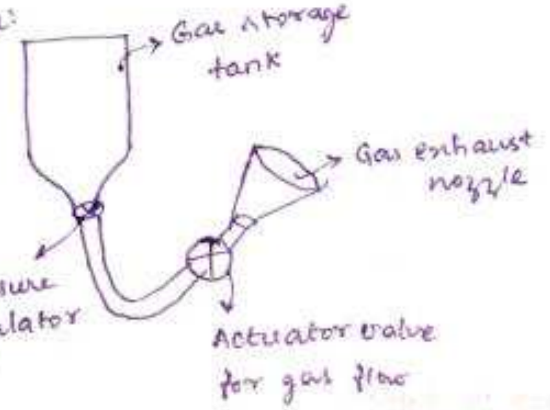
SSME:

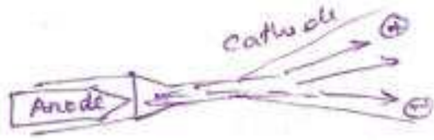
SSTO:

Different types of nozzles:

Types of propulsion systems:

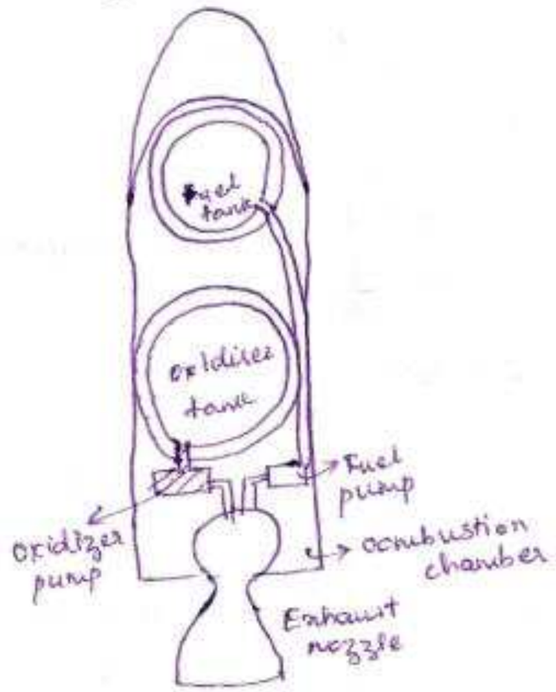
Thruster:

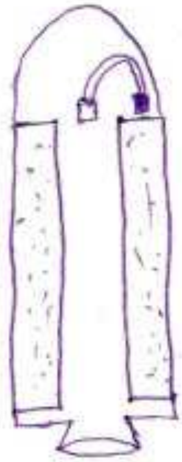




(no combustion for producing thrust)

Solid rockets:





Hybrid rocket motors:

* Mixture ratio strongly influenced by port design, fuel burn is generally incomplete, lower effective mass fraction.

* Low regression rates compared to solids, allows controlled imp but requires bigger port area for given thrust

* Insensitive to cracks or fuel grain aging, almost infinite storability of fuel grain.

Fuels:

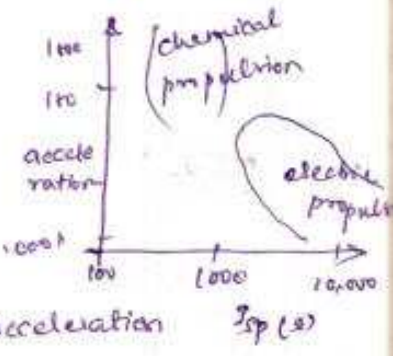
Hydroxyl terminated butadiene $(C_4H_6)(OH)_2$

Plexiglass - polymethane-methacrylate PMMA $(C_5H_8O_2)$

Oxidizers: Lox (O_2) - higher I_{sp}, dangerous to handle
- limited storability

Nitrous oxide (N_2O)
- lower I_{sp}, safe for handling
- highly storable.

Hydrogen peroxide (H_2O_2)



Electric propulsion:

Electric power, $P = \frac{F I_{sp} \cdot g_0}{2\eta}$

$\uparrow I_{sp}$ means $\downarrow$ acceleration

Thrusters:

Electro-thermal: propellant gas is electrically heated and expanded in the nozzle.

ex. arc jets, resistojets.

Electro-static: propellant is ionized and resulting ions are accele through elec. potential.

ex. Hall effect, Kauffmann type

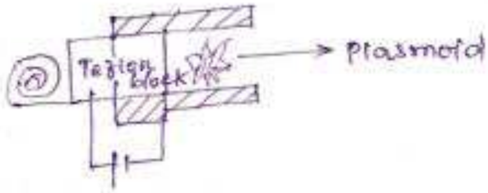
Electro-magnetic: both elec. & magnetic forces

ex: magnetoplasmadynamic thruster.

SPATER

Electrothermal arc jets:

Pulse plasma Thrusters: (PPT)



micro PPT-s

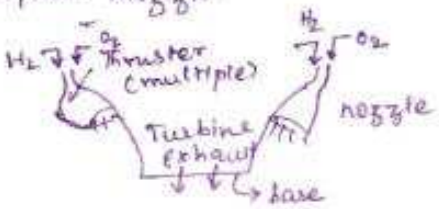
Ion-Engines:

Hall thrusters:

T : 5-400μN

Electrons emitted from cathode travel toward the anode

Aerospike Nozzle

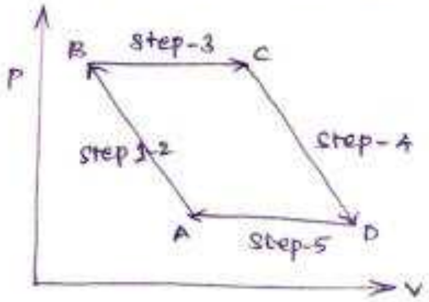


Telescope nozzle:

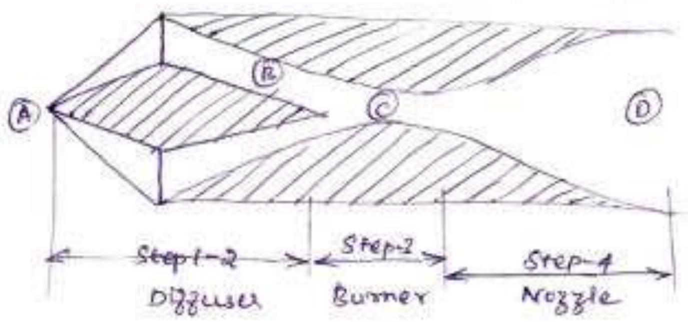
Disadv: Cooling
Manufacturing
Flight Experience

Ramjet Engines:

* Brayton cycle for Air breathing propulsion.



Step	Process
i) Intake (suck)	Isentrop comp
ii) Compression (squeeze)	adiab. Comp.
iii) Add heat (bang)	const. pr. combinati
iv) Extract work (blow)	Isentrop. expa. in no
v) Exhaust	heat exhaustion to m



Ideal Ramjet Cycle Analysis:

Net work available = work done in step 4 − work done in step 1-2

Net heat input = Heat input in Step 3 − Heat loss in exhaust

$$\text{Ideal cycle efficiency} = \frac{\text{net work output}}{\text{net heat input}}$$

$$\frac{\text{net work}}{\dot{m}} = (h_A - h_B) + (h_C - h_D)$$

$$\frac{\text{net } \cancel{\text{work}} \text{ heat input}}{\dot{m}} = (h_C - h_B)$$

$$\eta = \frac{\text{net work}/\dot{m}}{\text{net heat input}/\dot{m}}$$

$$\eta = 1 - \frac{h_D - h_A}{h_C - h_B}$$

$$h = c_p T$$

$$\eta = 1 - \frac{c_{p_g} T_D - c_{p_a} T_a}{c_{p_g} T_c - c_{p_a} T_b}$$

$c_{pa} \simeq c_{pg} \Rightarrow \quad \eta = 1 - \frac{T_D - T_A}{T_C - T_B}$

SPATER

$$\eta = 1 - \frac{\frac{T_D}{T_C} - \frac{T_A}{T_C} \times \frac{T_B}{T_B}}{1 - T_B/T_C}$$

$\Rightarrow$ C-D $\Rightarrow$ flow is isentropic.

$$\frac{T_D}{T_C} = \left(\frac{P_D}{P_C}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\eta = 1 - \frac{\left\{\left(\frac{P_D}{P_C}\right)^{\frac{\gamma-1}{\gamma}} - \frac{T_A}{T_B} \cdot \frac{T_B}{T_C}\right\}}{1 - T_B/T_C}$$

across diffuser,

$$\frac{P_A}{P_B} = \frac{P_A}{P_{0A}} \cdot \frac{P_{0A}}{P_{0B}} \cdot \frac{P_{0B}}{P_B}$$

$$= \left\{\frac{T_A}{T_{0A}}\right\}^{\gamma/\gamma-1} \cdot \left\{\frac{T_{0B}}{T_B}\right\}^{\gamma/\gamma-1} \cdot \frac{P_{0A}}{P_{0B}}$$

$$T_{0A} = T_{0B}$$

$$= \frac{P_{0A}}{P_{0B}} \times \left\{\frac{T_A}{T_B}\right\}^{\gamma/\gamma-1}$$

$$\eta = 1 - \frac{\left\{\left(\frac{P_D}{P_C}\right)^{\frac{\gamma-1}{\gamma}} - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \cdot \left(\frac{P_{0B}}{P_{0A}}\right)^{\frac{\gamma-1}{\gamma}} \frac{T_B}{T_C}\right\}}{1 - T_B/T_C}$$

Ideal burner: $P_C = P_B$

Ideal nozzle: $P_A = P_D$

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left\{1 - \left(\frac{P_{0B}}{P_{0A}}\right)^{\frac{\gamma-1}{\gamma}} \cdot \frac{T_B}{T_C}\right\}}{1 - T_B/T_C}$$

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}}$$

As combustor temperature difference goes up, efficiency goes up. As inlet pressure ratio goes down, which means stagnation pressure loss goes up, efficiency goes down.

Qn: Inlet mach no: 4 ; At inlet, a normal shock wave at dome, an oblique shock with $\beta = 40^\circ$; $\gamma = 1.4$; compare M_B and P_0 behind normal shock wave.

Qn: Molecular weight: ~~ϕ~~28.96 kg/mol-kg ; $R_g = 287$ J/kgK ; $P_\infty = 19.3$

$q = \frac{\dot{q}}{\dot{m}} = 500$ kJ/kg ; $T_\infty = 216.65K$; $\gamma = 1.4$. m_f is negligible

Stagnation enthalpy ~~and~~ of air & burned products = enthalpy air entering the combustor + heat released by chemical react

Sensible enthalpy of

$T_{02}, T_{03}, f, C_{p_2}, C_{p_3}, q_r$

$$\dot{m}_{air} ~~*~~(1+f)\, C_{p_3} T_{03} = \dot{m}_{air} C_{p_2} T_{02} + \dot{m}_{fuel} \cdot q_R$$

$$f = \frac{\frac{C_{p_3} T_{03}}{C_{p_2} T_{02}} - 1}{\frac{q_r}{C_{p_2} T_{02}} - \frac{C_{p_3} T_{03}}{C_{p_2} T_{02}}}$$

equivalence ratio $\phi = \frac{f_{actual}}{f_{stoichometric}}$

(used to characterize the mixture ~~mi~~ ratio of air-breathing engines)

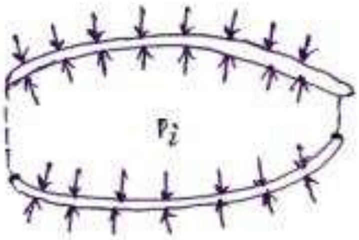
$\phi > 1 \Rightarrow$ rich $\qquad \phi < 1 \Rightarrow$ lean $\qquad \phi = 1 \Rightarrow$ stoichometric (no O_2 is unburned)

Qn: Calculate combustion of H_2 as fuel? What is stoichometric fuel ratio.

$$2H_2 + O_2 \rightarrow 2H_2O$$

$$\frac{\dot{m}H_2}{\dot{m}O_2}\Big|_{sto} = \frac{4 \times 1}{2 \times 16 \cdot 4} = \frac{1}{8} = ~~.125~~$$

$$(f)_{stoi} = \left\{\frac{\dot{m}H_2}{\dot{m}air}\right\}_{stoi} = \left\{\frac{\dot{m}H_2}{\dot{m}O_2}\right\}_{stoi} \cdot \frac{0.21\, \dot{m}O_2}{}$$



SPATER

$$-\iint_{C.S.} p\,d\vec{s} = \iint_{C.S.} \{\rho\vec{v}\cdot d\vec{s}\}\vec{v}$$

$$\int_{wall} P_{wall}\,dA_{wall} + P_\infty A_i - P_e A_e = \rho_e V_e^2 A_e - \rho_i V_i^2 A_i$$
$$= \dot{m}_e v_e - \dot{m}_i v_i$$

$\{P_e A_e - P_\infty V_\infty\}$ adding to both the sides

$$\int_{wall} P_{wall}\,dA_{wall} + P_\infty A_i - P_e A_e + \{P_e A_e - P_\infty V_\infty\} = \dot{m}_e v_e - \dot{m}_i v_i + \{P_e v_e - P_\infty V_\infty\}$$

Rocket Thrust equation:

$$\text{Thrust} = \dot{m} V_{exit} + A_{exit}\{P_{exit} - P_\infty\}$$

$$\frac{\text{Thrust}}{P_0 \cdot A_{throat}} = \frac{\dot{m} V_{exit}}{P_0 \cdot A_{throat}} + \frac{A_{exit}}{A_{throat}} \frac{\{P_{exit} - P_0\}}{P_0}$$

For choked throat, $\dfrac{\dot{m}}{A^* P_0} = \dfrac{1}{\sqrt{T_0}} \sqrt{\dfrac{\gamma}{R_g} \cdot \left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \Rightarrow \dfrac{\text{Thrust}}{P_0 A^*}$

$$\frac{\text{Thrust}}{P_0 \cdot A^*} = \frac{V_{exit}}{\sqrt{T_0}} \cdot \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} + \frac{A_e}{A^*}\left\{\frac{P_{exit} - P_\infty}{P_0}\right\}$$

For isentropic flow,

$$V_{exit} = \sqrt{2C_p(T_{0exit} - T_{exit})} = \sqrt{2C_p T_{0exit}}\left\{1 - \frac{T_{exit}}{T_{0exit}}\right\}^{1/2}$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1} \Rightarrow \frac{T_{exit}}{T_{0exit}} = \left\{\frac{P_{exit}}{P_{0exit}}\right\}^{\frac{\gamma-1}{\gamma}}$$

$$V_{exit} = \sqrt{2C_p T_{0exit}}\left\{1 - \left(\frac{P_{exit}}{P_{0exit}}\right)^{\frac{\gamma-1}{\gamma}}\right\}^{1/2}$$

$$\frac{\text{Thrust}}{P_0 A^*} = \frac{\sqrt{2C_p T_{0exit}}}{\sqrt{T_0}}\left\{1 - \left(\frac{P_e}{P_{0e}}\right)^{\frac{\gamma-1}{\gamma}}\right\}^{1/2}\sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} + \frac{A_e}{A^*}\left\{\frac{P_{exit} - P_\infty}{P_0}\right\}$$

Specific Impulse:

Cryogenic:

Chemical thrusters:

* Cold gas → No combustion, N_2H_4
* Monopropellant

- an unstable chemical that will decompose exothermically in the presence of a catalyst.

- chemical needs to unstable but not too unstable.

- V_2 used H_2O_2

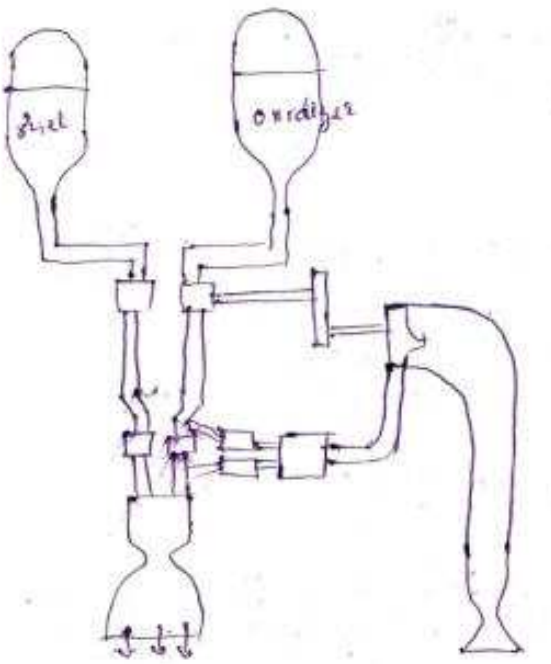
Bi-propellant system: - first stage liquid rockets

- 450 I_{sp}.

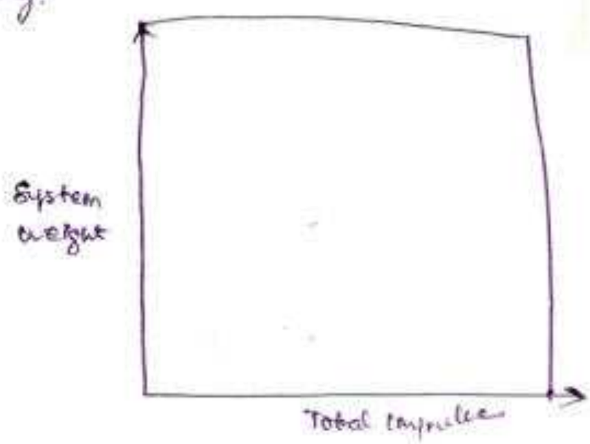
Turbine fed Bi-prop system:

(tested) (w/ NASA) → [illegible]

Rmin → SST



Rocket Nozzle cooling:



Propellant comparisions:

$$V_{exit} = \sqrt{2C_p(T_{0exit} - T_{exit})} = \sqrt{2C_pT_{0exit}}\left\{1-\left(\frac{P_e}{P_{0e}}\right)^{\frac{\gamma-1}{\gamma}}\right\}^{1/2}$$

$$\frac{Thrust}{P_0 A^*} = \frac{\sqrt{2C_pT_{0exit}}\left\{1-\left(\frac{P_e}{P_{0e}}\right)^{\frac{\gamma-1}{\gamma}}\right\}^{1/2}}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} + \frac{A_{exit}}{A^*}\frac{(P_{exit}-P_\infty)}{P_0}$$

$$= \left\{1-\left(\frac{P_e}{P_{0e}}\right)^{\frac{\gamma-1}{\gamma}}\right\}^{1/2}\sqrt{\frac{2C_p\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}} + \frac{A_{exit}}{A^*}\left\{\frac{P_{exit}-P_\infty}{P_0}\right\}$$

$$\frac{2C_p\gamma}{R_g} = \frac{2C_p\gamma}{(C_p-C_v)} = \frac{2\gamma}{1-1/\gamma} = \frac{2\gamma^2}{(\gamma-1)}$$

Isentropic nozzle, $P_{0exit} = P_0$

$$\frac{Thrust}{P_0 A^*} = \gamma\sqrt{\frac{2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\left\{1-\left(\frac{P_{ex}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right\}^{1/2} + \frac{A_{exit}}{A^*}\frac{(P_e-P_\infty)}{P_0}$$

$\underline{C^*}$

$$\frac{\dot m_e V_e}{P_0 A^*} = \frac{Thrust}{P_0 A^*} - \frac{A_e}{A^*}\frac{(P_e-P_\infty)}{P_0}$$

$$= \gamma\sqrt{\frac{2}{(\gamma-1)}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\left\{1-\left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right\}^{1/2}$$

nozzle expand until, $1 \gg \left\{\frac{P_{exit}}{P_0}\right\}^{\frac{\gamma-1}{\gamma}}$

$$\left.\frac{\dot m_{exit}V^*_{exit}}{P_0 A^*}\right|_{\text{infinite exp.}} = \gamma\sqrt{\frac{2}{(\gamma-1)}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$C^* = \left\{\frac{P_0 A^*}{\dot m_{exit}}\right\} = \frac{V^*_{exit}\left(\frac{\gamma-1}{2}\right)}{\gamma\sqrt{\frac{2}{(\gamma-1)}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}}$$

$$\frac{\dot m}{A^* P_0} = \frac{1}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}}$$

$$C^* = \frac{\sqrt{\gamma R_g T_0}}{\gamma\sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}} = \frac{C_0}{\gamma\sqrt{(2/\gamma+1)^{\gamma+1/\gamma-1}}}$$

$$C^* = \frac{C_0}{\gamma\sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}} = \frac{\sqrt{\gamma R_u}}{\gamma\sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}}\cdot\sqrt{\frac{T_0}{M_w}}$$

$$I_{sp} = \frac{Thrust}{\dot m g_0} = \frac{P_0 A^*}{\dot m g}\left\{\gamma\sqrt{\frac{2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\left(1-\left(\frac{P_e}{P_{0e}}\right)^{\frac{\gamma-1}{\gamma}}\right) + \frac{A_{ex}}{A^*}\frac{(P_e-P_\infty)}{P_0}\right\}$$

$$(I_{sp})_{ideal} = \frac{c^*}{g_0}\cdot\left\{\gamma\sqrt{\frac{2}{(\gamma-1)}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\right\} =$$

$$= \frac{1}{g_0}\cdot\sqrt{\frac{2\gamma R_u}{(\gamma-1)}}\cdot\sqrt{\frac{T_0}{M_w}}$$

Thrust co-efficient. $C_F = \frac{C_e}{c^*} = \frac{\text{Thrust}/\dot m}{P_0 A^*/\dot m}$

$\frac{A_e}{A^*}$ is a function of P_0/P_{exit}

$$\frac{A_e}{A^*} = \frac{1}{M_{exit}}\left\{\left(\frac{2}{(\gamma+1)}\right)\left(1+\frac{\gamma-1}{2}M_{exit}^2\right)\right\}^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$M_{exit} = \sqrt{\frac{2}{(\gamma-1)}\cdot\left\{\frac{P_0}{P_{ex}}^{\frac{\gamma-1}{\gamma}}-1\right\}}$$

$$\frac{A_{exit}}{A^*} = \sqrt{\frac{(2/\gamma+1)^{\gamma+1/\gamma-1}}{(2/\gamma-1)}}\cdot\sqrt{\frac{(P_0/P_{ex})^{\gamma-1/\gamma}}{(P_0/P_{ex})^{\gamma-1/\gamma}-1}}$$

$$\frac{\text{Thrust}}{P_0\cdot A^*} =$$

C_F is a fn. of (i) combustion process (P_0, γ)

(ii) Nozzle expansion. P_{exit}

(iii) Back pressure (P_∞)

$$\frac{P_0}{P} = \left\{1+\frac{\gamma-1}{2}M_{exit}^2\right\}^{\gamma/\gamma-1}$$

$$\left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}} = \left\{1+\frac{\gamma-1}{2}M_{exit}^2\right\}$$

$$\frac{\text{Thrust}}{P_0 A^*} = \gamma\sqrt{\frac{2}{(\gamma-1)}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}\left\{1-\left(\frac{P_{ex}}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right\}^{1/2} + \frac{(P_e-P_\infty)}{P_0}\cdot\sqrt{\frac{(2/\gamma+1)^{\gamma+1/\gamma-1}}{2/\gamma-1}}\cdot\sqrt{\frac{(P_0/P_{ex}}{(P_0/P_{ex}}}$$

$$C_F = \frac{\text{Thrust}/\dot m}{P_0 A^*/\dot m}$$

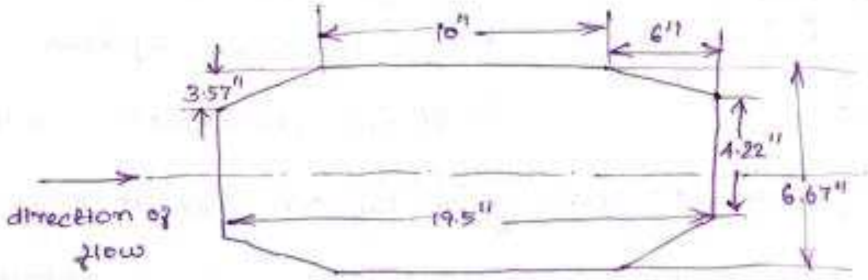
$$C_F = \gamma\sqrt{\frac{2}{(\gamma-1)}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}}\left\{1-\frac{P_e}{P_0}^{\frac{\gamma-1}{\gamma}}\right\}^{1/2} + \frac{(P_e-P_\infty)}{P_0}\cdot\sqrt{\frac{(2/\gamma+1)^{\gamma+1/\gamma-1}}{2/\gamma-1}}\cdot\sqrt{\frac{(P_0/P_{ex})}{(P_0/P_{ex})}}$$

$$\frac{A_{exit}}{A^*} = \frac{\frac{2}{n+1}\left\{1 + \frac{n-1}{2} M_{exit}^2\right\}^{n+1/2(n-1)}}{\sqrt{\frac{2}{n-1}\left\{(P_o/P_{ex})^{n-1/n} - 1\right\}}}$$

$$= \sqrt{\frac{(2/n+1)^{n+1/n-1}}{(2/n-1)}} \sqrt{\frac{(P_o/P_{ex})^{n-1/n}}{(P_o/P_{ex})^{n-1/n} - 1}}$$

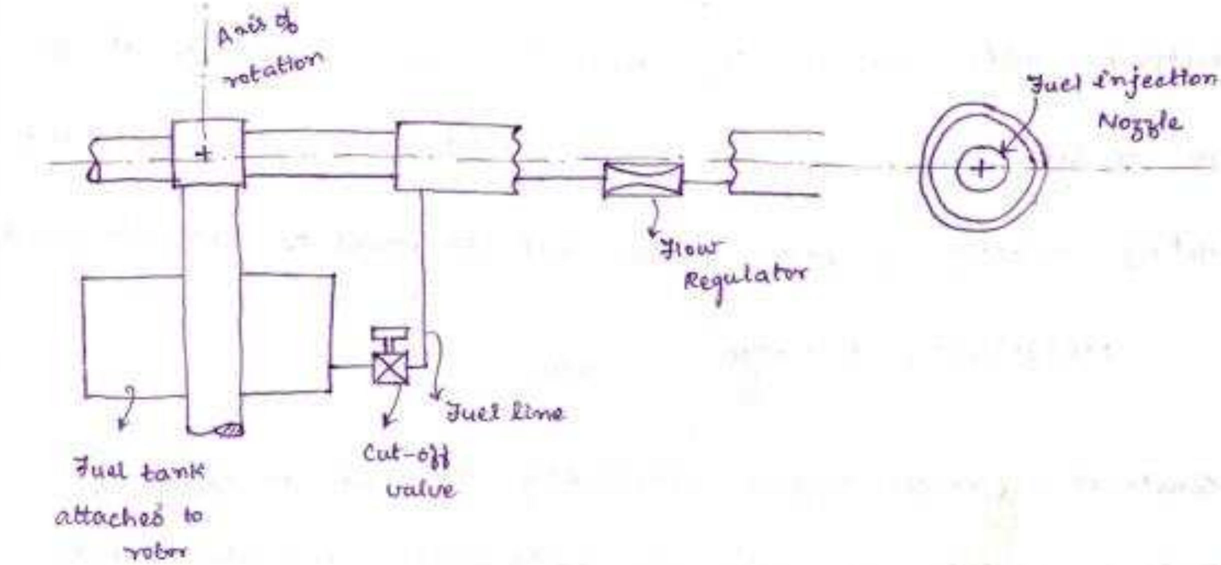
Subsonic Ramjet:

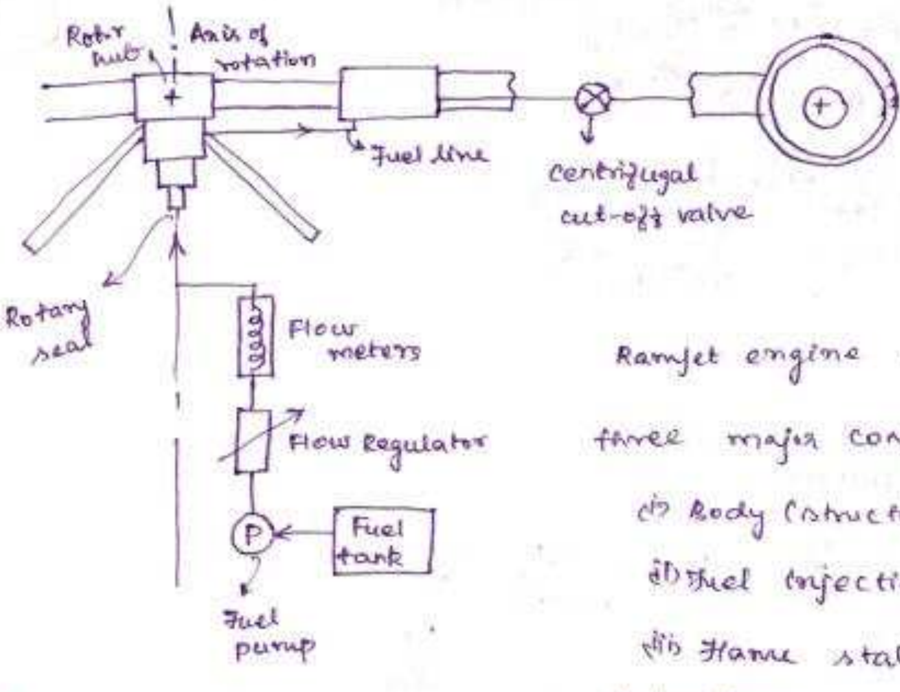
Body structure (Diffuser):



The body of a subsonic ramjet engine is an open duct composed of a divergent nozzle (diffuser), a combustion chamber and a convergent nozzle (nozzle).

Fuel injection system:





Ramjet engine is composed of three major components.

(i) Body (structure)

(ii) Fuel injection system

(iii) Flame stabilisation system

The amount of thrust ~~desion~~ ~~governing~~ derived from the ramjet engine at the given speed determines the size of diffuser entrance area. The larger diffuser entrance area, the greater the thrust. A designer decides to design a ramjet engine a small ramjet helicopter which requires engines at rotor tips at 25 lb per engines. The helicopter is to be equipped with rotors 23 ft in dia. and operate at rpm of 665. since engines are attached to the rotor, operating velocity of each engine will be equal to rotor tip

$$RTV\,(ft/sec) = \frac{\pi \times D \times RPM}{60} \longrightarrow (1)$$

To construct a ramjet engine producing T = 25 lb at operating velocity of 800 ft/s. The diffuser entrance should have $10\,in^2$ area = 800. The ratio of diffuser entrance area to diffuser exit area varies normally from 3-4. The length of diffuser depends on designer's choice. A hollow cone frustum with a curve insert may be used for commercial ramjet designs because its shorter leng offers less drag. curved insert allows the use of larger cone angle. The efficiency of diffuser will fall below an acceptable limit. Diffuser length for ramjet can be obtained. **SPATER**

Diffuser length = Diffuser diameter at exit − diffuser diameter at entrance

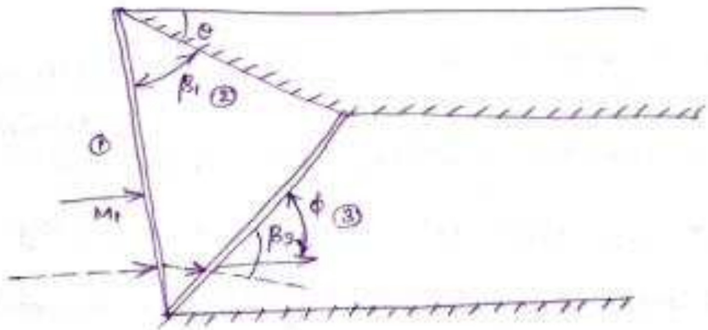
Combustion chamber is a hollow cylinder with a diameter exit equal to diffuser exit diameter. It's length is usually determined by trial & error. A good rule of thumb is make the combustion chamber length to be three times than the diffuser entrance diameter. Nozzle is located downstream from the combustion chamber. Has its ~~coor~~ diameter equal to combustion chamber. Nozzle exit diameter is determined by the temperature in the combustion chamber. Make the nozzle exit area 1.4 times than greater than the diffuser entrance area.

The recommended material for the fabrication of ~~stain~~ Type 310, 312, 317 stainless steel, A sheet thickness of 16 gauge upto a max. diameter of 7". If a base plate is employed in the body structure, 20 gage sheet stock may be used to fabricate combustion chamber, diffuser and exhaust nozzle.

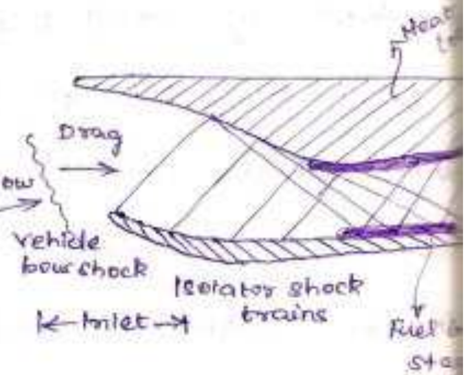
commercial ~~nozzles are~~ ramjet shells are manufactured by stamping, spinning, etc. The recommended commercial procedure is to roll the combustion chamber is ~~to roll the~~ from sheet stock into a hollow cylinder and diffuser and nozzles into cone frustums and join all three parts by welding. A smooth finish inside the engine is important.

Scramjet:

- we need to keep the flow path supersonic.



$M = 3.6$; $\theta = 20^\circ$; $\gamma = 1.4$



$$\tan\theta = \frac{2\{M_1^2 \sin^2\beta - 1\}}{\tan\beta\{2 + M_1^2(\gamma + \cos 2\beta)\}}$$

$\beta_1 = 34.11^\circ$

$M_{1,n} = M_1 \sin\beta_1 = 2.3 \sin 34.11^\circ$

$= 2.018$

Normal shock wave, $M_2 = 2.35$

$\beta_2 = 45.05^\circ$

$\phi =$

$M_3 = 1.48^\circ$

Ramp shock # oblique shock # Normal shock

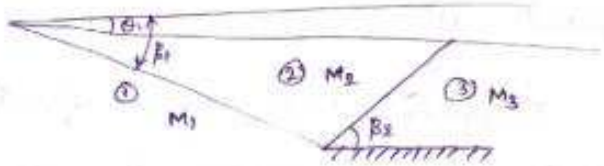
Air enters at Mach 10 at 30 km altitude.

$P_\infty = 1.1718$ kPa ; $T = 226.65$ K ; $\theta_{ramp} = 4^\circ$;

H_2 as fuel : $\phi = 1$; $\gamma = 1.1565$; 24.519

Combustion chamber : $P_3 = 6.35$ kPa

$T_3 = 381.47$ kPa



$a = \sqrt{\gamma RT} = \sqrt{1.565 \times 226.65 \times 24.519}$

$= 93.258$ m/s

Conditions at 2:

$$\tan\theta = \frac{2\{M_1^2 \sin^2\beta_1 - 1\}}{\tan\beta\{2 + M_1^2(\gamma + \cos 2\beta)\}}$$

$\beta_1 = 8.6531^\circ$; $M_2 = 8.6227$

$P_2/P_\infty = 2.4741$; $T_2/T_\infty = 1.323$

$P_{02}/P_{0\infty} = 0.9283$

$\beta_1 = 8.65^\circ$; $\beta_2 = 9.521^\circ$

$M_2 = 8.622$; $M_3 = 7.576$; $P_3/P_\infty = 5.496$

$P_2/P_\infty = 2.47$; $P_3/P_2 = 2.2$; $P_{03}/P_{0\infty} = .8832$

$T_2/T_\infty = 1.32$ $\qquad$ $P_{02}/P_{0\infty} = 0.928$

$T_3/T_2 = 1.27$ $\qquad$ $P_{03}/P_{02} = 0.951$

$T_3/T_\infty = 1.68$ $\qquad$ $P_3 = 6.56$ kPa ; $T_3 = 381.46$ K.

Liquid Rockets:

* Combustion produces high temperature gaseous products.
* Gases escape through nozzle throat
* Nozzle throat chokes (maximum mass flow)
* $$\frac{\dot{m}}{A^*} = \frac{\gamma}{R_g}.$$

Gaseous mass trapped in chamber: $\frac{\partial}{\partial t}(M_c) = (\dot{m}_{fuel} + \dot{m}_{ox}) - \dot{m}_{nozzle}$

Assuming constant flame temperature,

$$\rho_c = \frac{P_c}{R_g T_0} \rightarrow \frac{\partial}{\partial t}(\rho_c) \simeq \frac{1}{R_g T_0} \cdot \frac{\partial}{\partial t}(P_0)$$

$$\frac{\partial P_c}{\partial t} \cdot \frac{V_c}{R_g T_0} + \frac{P_c}{R_g T_0} \cdot \frac{\partial V_c}{\partial t} = (\dot{m}_{fuel} + \dot{m}_{ox}) -$$

Apply liquid rocket analysis, $\frac{1}{V} \cdot \cancel{\frac{\partial V_c}{\partial t}} \rightarrow 0$

$$\frac{\partial P_0}{\partial t} + P_0 \left\{ \frac{A^*}{V_c} \sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \right\} = \frac{R_g T_0}{V_c} (\dot{m}_{fuel} + \dot{m}_{ox})$$

Combustion chamber scaling:

$$V_c = A_t \left\{ L_c \epsilon_c + \frac{1}{3}\sqrt{\frac{A_t}{\pi}} \cdot \frac{(\epsilon_c - 1)}{\tan\theta} \right\}$$

optimum t_c must be determined experimentally.

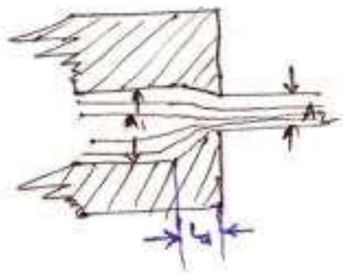
SPATER

LOX-Kerosene

Space Propulsion Analysis & Design - Ronald W. Humble

$$V_c = A^* \left\{ t_c \epsilon_c + \frac{1}{3}\sqrt{\frac{A^*}{\pi}} \cdot \frac{(\epsilon - 1)}{\tan\theta} \right\}$$

Injector Geometry:



Discharge co-efficient:

$$C_d = \frac{C_v}{\sqrt{1-(A_2/A_1)^2}}$$

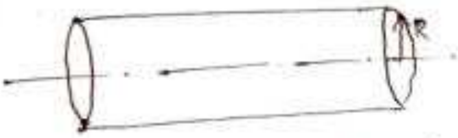
volumetric flow is defined as,

$$Q_v = A_2 V_{2actual} = A_2 C_d \sqrt{\frac{2(P_1-P_2)}{\rho}}$$

Finally, mass flow is,

$$\dot{m} = \rho Q_v = A_2 C_d \sqrt{2\rho(P_1-P_2)}$$

$$= \frac{A_2 \cdot C_v \cdot \sqrt{2\rho(P_1-P_2)}}{\sqrt{1-(A_2/A_1)^2}}$$



Equate terms,

$$Q_v = -\frac{\pi R^4}{8\mu} \cdot \frac{\partial P}{\partial x}$$

$$C_d = \frac{D^2}{32 \mu L_d} \sqrt{\frac{\rho}{2}(P_1-P_2)}$$

$$\frac{1}{2} \cdot \frac{\rho V_{actual}}{C_d} = \sqrt{\frac{\rho}{2}(P_1-P_2)}$$

$$C_d = \frac{1}{8} \cdot \sqrt{\frac{D}{L_d}} \sqrt{Re_d}$$

Collect terms,

$$C_d = \frac{1}{8} \cdot \frac{D}{L_d} \sqrt{Re_L}$$

Laminar discharge co-eff.

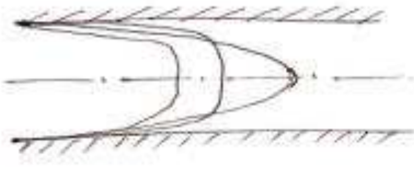
is - α → diameter of injector port

$\frac{1}{\alpha}$ → square root of injector port length

α → Reynold's number

→ from one working fluid to another

Injector Design for Turbulent flow:



laminar

Turbulent Flow: $\frac{\partial p}{\partial x} = -\frac{4\tau_0}{D} = -4 \cdot \frac{1}{2}\rho \bar{U}^2 \cdot \frac{C_f}{D}$

Injector Design for Turbulent Flow

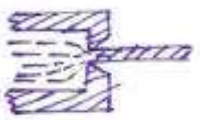
$$Q_D = \frac{-\pi D^4}{8\mu} \cdot \frac{1}{Re_D C_{fD}} \cdot \frac{\partial p}{\partial x}$$

$$\text{for: } V_{actual} = C_d \sqrt{\frac{2(P_1-P_2)}{\rho}}$$

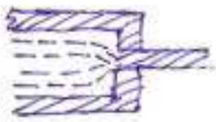
$$\frac{1}{2}\rho \frac{V_{actual}}{C_d} = \sqrt{(P_1-P_2)\frac{\rho}{2}}$$

Injector Discharge co-efficients:

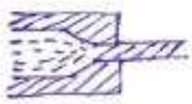
sharp edged orifice



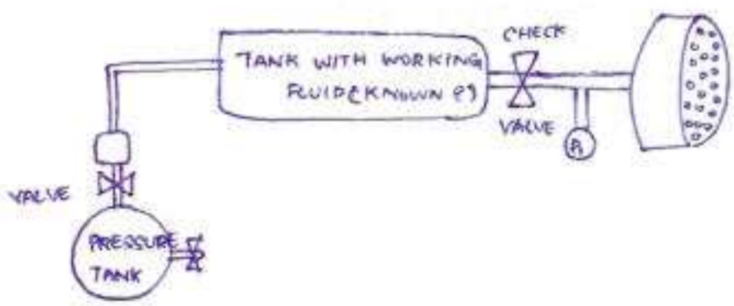
short tube with rounded entrance ($L/D > 3.0$)



Short tube with conical entrance



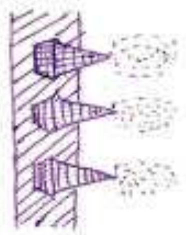
	Diameter (mm)	C_d
sharp edged orifice	Above 2.5	0.61
	Below 2.5	0.65
short tube with rounded entrance ($L/D > 3.0$)	1.00	0.88
	1.57	0.9
	1.00	0.7

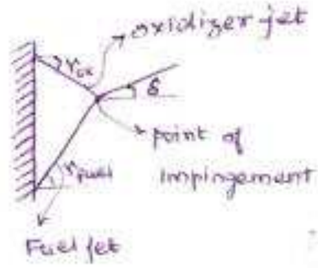


$$C_d = \frac{1}{8} \cdot \sqrt{\frac{D}{L_d}} \sqrt{Re_D}$$

$$\frac{(C_d)_{oxidizer}}{(C_d)_{test\ fluid}} = \frac{\sqrt{Re_{oxidizer}}}{\sqrt{Re_{test\ fluid}}} = \frac{\rho_{ox}\,\mu_{tf}}{[illegible]}$$

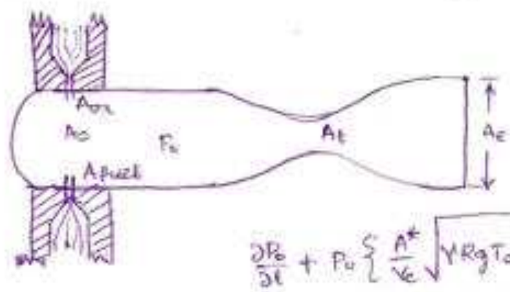
Biasius (dotted) ignition





Design criteria for injection angle.

$$\frac{\sin \gamma_{ox}}{\sin \gamma_f} = \frac{A_{fuel}}{A_{ox}} \cdot \frac{Cd_f^2}{Cd_{ox}^2} \cdot \frac{(P_f - P_0)}{(P_{ox} - P_0)}$$



$$\frac{\partial P_0}{\partial t} + P_0 \left\{ \frac{A^*}{V_c} \sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}} \right\} = \frac{R_g T_0}{V_c} \left[A_{fuel}\, Cd_f \sqrt{2\rho_f (P_f - P_0)} + \ldots \sqrt{2\rho_{ox}(P_{ox} \ldots} \right.$$

Mixture Ratio: $M_R = \frac{\dot{m}_{ox}}{\dot{m}_{fuel}}$

$$= \frac{A_{ox}\, Cd_{ox} \sqrt{2\rho_{ox}(P_{ox} - P_0)}}{A_f\, Cd_f \sqrt{2\rho_f (P_f - P_0)}}$$

$$\dot{m}_{ox} = \dot{m}_{fuel} \cdot M_R$$

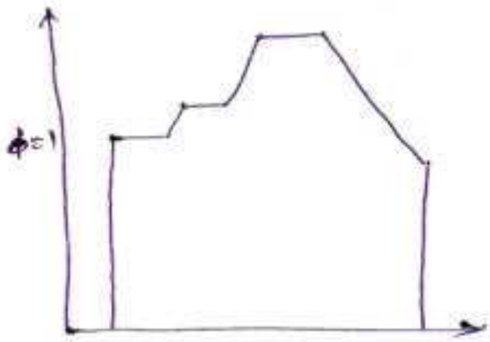
$$\frac{\partial P_0}{\partial t} =$$

Squaring on both the sides,

$$P_0^2 \left\{ \frac{A^*}{(1+M_R)} \right.$$

$$P_{0ss} = \frac{-1 \pm \sqrt{1 + 4P_f \left(\frac{A^*}{(1+M_R) A_f Cd_f}\right)^2 \left(\frac{\gamma}{2\rho_f R_g T_0} \left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}\right)}}{}$$

Mass flow ent

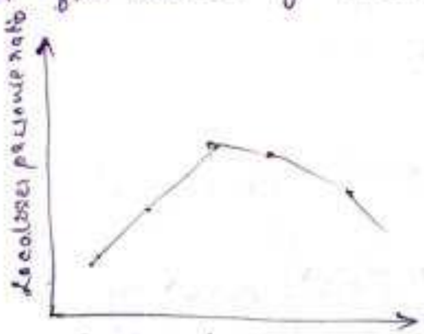


Design of scramjet is much abt minimizing drag aas maximizing thrust.

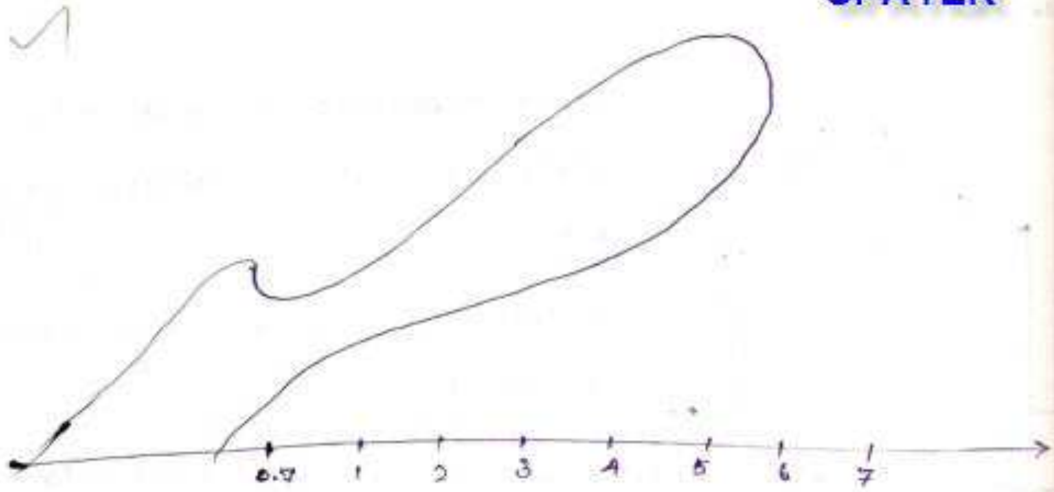
Integration of engine into airframe is the key.

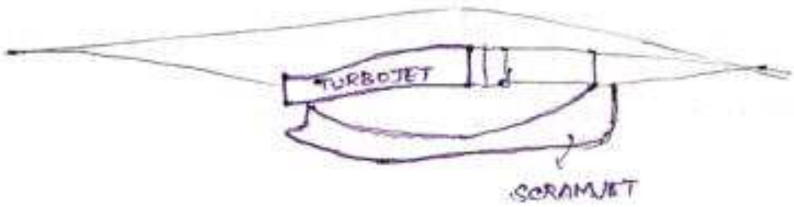
Supersonic combustion cannot maintain stable combustion below Mach ~6.

Scramjets are feasible only for achieving hypersonic speeds not for achieving them from zero speed.



Rocket Based combined cycle: (RBCC Mission profile)





$$\dot{q}_{LE} = H_{t\xi}\,[h_0 - h_{wall}]\left[\frac{1}{R_{LE}}\sqrt{\frac{2(P_{02}-P_\infty)}{\rho_{02}}}\right]^{1/2}\left[C_{p\infty}T_\infty + \frac{V_\infty^2}{2} - C_{p\,wall}T_{wall}\right]$$

$$\dot{T}_{wall} = \frac{(\phi(0)H_{t\xi})\left[C_pT_\infty + \frac{V_\infty^2}{2} - C_{p\,wall}T_{wall}\right] + \left[\frac{\alpha}{2}\sigma T_2^4 - \epsilon\sigma T_{wall}^4\right]}{\rho_{LE}\, C_{pLE}\, \tau_{LE}}$$

SCRAMJET Flight tests:

X-43A:

Hypersonic Physics - Propulsion:

* Natural and forced boundary layer transition.
* Turbulence
* Seperation caused by shock-boundary layer interaction
* catalytic wall effects.